LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2025**

PST1MC04 – SAMPLING THEORY

Date: 30-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Define the following**
a)	Fixed and Varying size sampling design.
b)	Lahiri's Method.
c)	Systematic Sampling.
d)	Multistage Sampling.
e)	Adaptive Sampling.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**Fill in the blanks**
a)	The probability that a specific unit is included in the sample is called its _____ probability.
b)	The Des Raj estimator is commonly used in _____ sampling, where the probability of selection is related to the size of the units.
c)	$\hat{Y}_{LSS}$ is more efficient than $\hat{Y}_{SRS}$, when the population values Y_i satisfy _____.
d)	The ratio estimator namely _____ can be used for estimating the population total Y only when X is known.
e)	In Warner's model, if the respondent uses a biased coin with probability (p) of heads, the expected proportion of "Yes" responses is given by _____.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Prove that unbiasedness of an estimator depends on the sampling design.
4	a. Describe the Simple Random Sampling Without Replacement (SRSWOR) design and explain the unit drawing mechanism. (4) b. In SRSWOR, show that (6) (i) $\pi_i = \frac{n}{N}$; $i = 1, 2, \ldots, N$, and

	(ii) $\pi_{ij} = \frac{n(n-1)}{N(N-1)}$; $i, j = 1, 2, \ldots, N$ and $i \neq j$.
5	Describe Random Group Method in detail. Also, demonstrate that an unbiased estimator of Y under random group method is $\hat{Y}_{RG} = \sum_{i=1}^{n} \frac{y_i}{x_i} T_x(i)$.
6	Derive V($\hat{Y}_{st}$) under Neyman allocation.
7	Explain Simmons Randomized Response Technique and find the estimated variance of $\hat{\pi}_A$ when π_Y is unknown.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Examine whether $T_1(s) = \frac{1}{n(s)} \sum_{i \in s} Y_i$ and $T_2(s) = [\max_{i \in s}\{Y_i\} + \min_{i \in s}\{Y_i\}] / 2$ are unbiased for $\bar{Y}$ under the sampling design $P(s) = \begin{cases} \frac{1}{4} & if\ n(s) = 3 \\ 0 & otherwise \end{cases}$. Given $Y_1 = 2$, $Y_2 = 3$, $Y_3 = 4$, and $Y_4 = 7$.
9	In PPSWOR sampling scheme, give the reason for using Desraj ordered estimator instead of Hurwitz – Thompson estimator. Verify if Desraj ordered estimator is unbiased for population total.
10	Under Stratified Random Sampling, show that an UBE of Y is $\hat{Y}_{st} = \sum_{h=1}^{L} \hat{Y}_h$. Also, obtain the formula for $\hat{Y}_{st}$, $V(\hat{Y}_{st})$ and $v(\hat{Y}_{st})$ under the design (i) SRSWOR and (ii) PPSWR .
11	Obtain Hartley – Ross unbiased ratio type estimator for population total.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Describe Hansen-Hurwitz Estimator and show that $\hat{Y}_{HHE}$ is unbiased for Y. Also, show that $v(\hat{Y}_{HHE}) = \frac{1}{n(n-1)} \sum_{i=1}^{n} \left(\frac{y_i}{p_i} - \hat{Y}_{HHE}\right)^2$.
13	Explain in detail Warner's Model and find the estimated variance of $\hat{\Pi}_A$.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	a. Explain Midzuno Sampling Design (MSD). (6) b. Under MSD, show that (7 + 7) i. the first order inclusion probability is $\Pi_i = \frac{N-n}{N-1} \cdot \frac{X_i}{X} + \frac{n-1}{N-1}$, i=1,2…, N, and ii. the second order inclusion probability is $\Pi_{ij} = \frac{(N-n)(n-1)}{(N-1)(N-2)} \cdot \frac{X_i + X_j}{X} + \frac{(n-1)(n-2)}{(N-1)(N-2)}$, $i \neq j = 1, 2, \ldots, N$.
15	Describe Regression Estimation and derive an approximate expression for the bias and MSE of $\hat{Y}_{LR}$.

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